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COMMENT

The triviality of the Abelian Thirring quantum field model

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Abstract. By using a Grassmannian polymer representation for the fermionic functional determinant we argue the triviality of the vectorial four-fermion interaction for spacetime with dimensionality greater than two.

One of the most interesting problems in D-dimensional Euclidean field theories is the appearance of a critical dimensionality where above this value the associated field theory becomes trivial [1, 2].

Our aim in this comment is to present the Parisi geometrical analysis [3] generalised to the fermionic case by analysing the critical spacetime dimension for the vectorial four-fermion interaction (the Abelian Thirring model).

Let us start our analysis by considering the Thirring model Euclidean partition functional in $\mathbb{R}^D$ with the fermionic fields integrated out:

$$Z[g]=\int DA_\mu \exp\left[-\frac{1}{2}\int dx^D A_\mu^2(x)\right] \det \not{D}(A_\mu) \tag{1}$$

where $\not{D}(A_\mu)\equiv\gamma_\mu(\partial_\mu+gA_\mu)$ is the Euclidean Dirac operator in the presence of the external auxiliary vectorial field and g is the bare theory's coupling constant.

We aim to show that $Z[g]=Z[g=0]$ when $D>2$ since this result will lead, formally at least, to be triviality of (1).

By using the fermionic loop representation for $\det \not{D}(A_\mu)$, as displayed in [4], we can write this functional determinant as a Grassmannian path integral:

$$\begin{aligned}\det \not{D}(A_\mu) &= \sum_{[\chi_\mu^{\mathrm{F}}(\xi,\theta)]} \exp\left(\int_0^1 d\xi \int d\theta\, A_\mu[\chi_\mu^{\mathrm{F}}(\xi,\theta)]\, D\chi_\mu^{\mathrm{F}}(\xi,\theta)\right)\\ &= \sum_{[\chi_\mu^{\mathrm{F}}(\xi,\theta)]} \int d^D x\, A_\mu(\chi) J_\mu^{\mathrm{F}}[x_\mu^{\mathrm{F}}(\xi,\theta)]\end{aligned} \tag{2}$$

where the sum $\Sigma_{[\chi_\mu^{\mathrm{F}}(\xi,\theta)]}$ is defined by (4) in [4] and $J_\mu^{\mathrm{F}}[\chi_\mu^{\mathrm{F}}(\xi,\theta)]$ is the current associated with the Grassmannian loop $\chi_\mu^{\mathrm{F}}(\xi,\theta)=\chi_\mu(\xi)+i\theta\psi_\mu(\xi)$ ($\theta^2=0$; $0\leqslant\xi\geqslant 1$). Through a g-power series expansion and integrating the Gaussian $A_\mu(\chi)$ functional integral we get, for instance, for its first coefficient $dz[g]/dg|_{g=0}=z_1$, the following expression:

$$\begin{aligned}z_1 = \sum_{[\chi_\mu^{\mathrm{F}}(\xi,\theta)]} \exp\frac{1}{2}\int_0^1 d\xi\, d\theta \int_0^1 d\xi'\, d\theta'\, D\chi_\mu^{\mathrm{F}}(\xi,\theta)\delta^{(D)}\\ \times(\chi_\mu^{\mathrm{F}}(\xi,\theta)-\chi_\mu^{\mathrm{F}}(\xi',\theta')\, D\chi^{\mathrm{F}}(\xi',\theta').\end{aligned} \tag{3}$$

We can understand (3) as the partition functional associated with a gas of closed polymers $[\chi^{\mathrm{F}}_{\mu}(\xi, \theta)]$ possessing a Grassmannian structure and interacting among themselves by a self-avoiding interaction $\delta^{(D)}[\chi^{\mathrm{F}}_{\mu}(\xi, \theta) - \chi_{\mu}(\xi', \theta')]$.

In order to argue the triviality of the fermionic polymer gas we follow Parisi [3] by assigning a Hausdorff dimension d_{H} for the 'set' $[\chi^{\mathrm{F}}_{\mu}(\xi, \theta);\ \theta^2 = 0;\ 0 \leqslant \xi \leqslant 1]$. A natural Hausdorff dimension for this set is given by the exponent of the fermion free field propagator in the momentum space which is 1, so $d_{\mathrm{H}}[\chi^{\mathrm{F}}_{\mu}(\xi, \theta)] = 1$.

By using now the geometrical intersection rule $d_{\mathrm{H}}(A \cap B) = d_{\mathrm{H}}(A) + d_{\mathrm{H}}(B) - D$ [3], with D being the spacetime dimensionality, we obtain that the support set of the self-avoiding interaction $[\delta^{(D)}(\chi_{\mu}(\xi, \theta) - \chi_{\mu}(\xi', \theta')]$ has a negative Hausdorff dimension for $D > 2$ which means that this set is empty.

As a consequence we have the analytical relation

$$\int_0^1 \mathrm{d}\xi\, \mathrm{d}\theta \int_0^1 \mathrm{d}\xi'\, \mathrm{d}\theta'\, \mathrm{D}\chi^{\mathrm{F}}_{\mu}(\xi, \theta)\delta^{(D)}(\chi^{\mathrm{F}}_{\mu}(\xi, \theta) - \chi^{\mathrm{F}}_{\mu}(\xi', \theta'))\, \mathrm{D}\chi^{\mathrm{F}}_{\mu}(\xi', \theta') = 0 \qquad (4)$$

which indicates, in turn, the triviality of the theory, since this argument can be straightforwardly applied for any arbitrary coefficient z_n and leading to the result $z_n = z_1$.

Finally we remark that by reformulating the Thirring theory in the loop space, we can in principle define the theory for any general manifold m as spacetime by including the constraint $[\chi^{\mathrm{F}}_{\mu}(\xi, \theta)] \subset m$ in the path integral (3). Note that m may be fluctuating [5].

Work in this direction is in progress and will appear elsewhere.

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